Mon. Not. R. Astron. Soc. **000**, 1–7 (2004) Printed 2 February 2008 (MN LaTeX style file v2.2)

Cosmic acceleration and extra dimensions: constraints on modifications of the Friedmann equation

Øystein Elgarøy[1,2⋆] and Tuomas Multamäki[2]†
[1]Institute of theoretical astrophysics, University of Oslo, Box 1029, 0315 Oslo, Norway
[2]NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

2 February 2008

ABSTRACT
An alternative to dark energy as an explanation for the present phase of accelerated expansion of the Universe is that the Friedmann equation is modified, e.g. by extra dimensional gravity, on large scales. We explore a natural parametrization of a general modified Friedmann equation, and find that the present supernova type Ia and cosmic microwave background data prefer a correction of the form $1/H$ to the Friedmann equation over a cosmological constant. We also explore the constraints that can be expected in the future, and find that there are good prospects for distinguishing this model from the standard cosmological constant to very high significance if one combines supernova data with a precise measurement of the matter density.

Key words: cosmology:theory – cosmology:observations – cosmological parameters

1 INTRODUCTION

There is mounting evidence that we are living in a universe dominated by a dark energy component, acting as a source of gravitational repulsion causing late-time acceleration of the expansion rate. Early hints came from the classical test of using the magnitude-redshift relationship with galaxies as standard candles (Solheim 1966), but the reality of cosmic acceleration was not taken seriously until the magnitude-redshift relationship was measured recently using high-redshift supernovae type Ia (SNIa) (Riess et al. 1998; Perlmutter et al. 1999). Cosmic acceleration requires a contribution to the energy density with negative pressure, the simplest possibility being a cosmological constant. Independent evidence for a non-standard contribution to the energy budget of the universe comes from e.g. the combination of the cosmic microwave background (CMB) and the large-scale structure (LSS) of the Universe: the position of the first peak in the CMB is consistent with the universe having zero spatial curvature, which means that the energy density is equal to the critical density. Since observations of the LSS show that the contribution of standard sources of energy density, whether luminous or dark, is only a fraction of the critical density, an extra, unknown component is needed to account for the spatial flatness of the Universe (Efstathiou et al. 2002; Tegmark et al. 2003).

The simplest explanation for the present accelerated phase of expansion is to re-introduce Einstein's cosmological constant, Λ. The resulting model with Λ, baryons, radiation, and cold dark matter (CDM) is consistent with all large-scale cosmological observations like the anisotropies in the CMB radiation and the power spectrum of galaxies (Tegmark et al. 2003). However, the value of Λ implied by the observations is tiny compared to the value inferred from the fact that it quantifies the energy of the vacuum in quantum field theory. Faced with this problem, the popular choice is to set Λ to zero and invoke a new component to explain the acceleration. This does not solve the problem of the smallness of Λ; if it is indeed equal to zero, one still needs to understand the physical mechanism behind this. Nevertheless, one may hope that $\Lambda = 0$ may be easier to explain than a small, but non-zero Λ. The question is then what the unknown component driving the accelerated phase of expansion is. One needs to introduce a component with negative pressure, and this can be done e.g. by invoking a slowly evolving scalar field (Wetterich 1988; Peebles & Ratra 1988; Ratra & Peebles 1988), or a negative-pressure fluid, e.g. a Chaplygin gas (Kamenshchik, Moschella & Pasquier 2001; Bilic, Tupper & Viollier 2001). A negative-pressure fluid, however, can be problematic due to its fluctuations. Fluctuations of the unknown fluid can grow very rapidly and hence LSS surveys can place strict constraints on such models, see e. g. Bean & Dore (2003); Sandvik et al. (2002). In order to circumvent this, it is sometimes assumed that the new fluid does not fluctuate on the scales of interest and its existence is visible only through modified background evolution.

A different point of view, which we follow here, is that

⋆ E-mail: oelgaroy@astro.uio.no
† E-mail:tuomas@nordita.dk

the late time acceleration is not due to an unknown component but rather that the Friedmann equation is modified on large scales, e.g. due to extra dimensional physics.

The structure of this paper is as follows. In section 2 we motivate the modified Friedmann equation we discuss in later sections. In section 3 consider fits to SNIa data, and in section 4 we extend the fits to include CMB data from the Wilkinson Microwave Anisotropy Probe (WMAP). In section 5 we discuss future prospects for constraining the form of the modified Friedmann equation using SNIa data, and section 6 contains our conclusions.

2 MODIFIED FRIEDMANN EQUATION

We will consider a modification to the Friedmann equation with no curvature in the spirit of Dvali & Turner (2003), where they consider

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\rm m}(1+z)^3 + (1-\Omega_{\rm m})\left(\frac{H}{H_0}\right)^\alpha, \qquad (1)$$

where α is a parameter restricted to be less than 2 from Big Bang Nucleosynthesis (BBN) considerations. One can motivate Friedmann equations of this form as general parametrizations of the leading effect arising from modified gravity theories. As an example, consider a simple, single extra-dimensional model (Dvali, Gabadadze & Porrati 2000; Deffayet 2001; Deffayet, Dvali & Gabadadze 2002). The effective, low-energy action is given by

$$S = \frac{M_{\rm Pl}^2}{r_{\rm c}}\int d^4x dy \sqrt{g^{(5)}}\mathcal{R} + \int d^4x\sqrt{g}(M_{\rm Pl}^2 R + \mathcal{L}_{\rm SM}), \qquad (2)$$

where $M_{\rm Pl}^2 = 1/8\pi G$, G is Newton's gravitational constant, $g_{AB}^{(5)}$ is the 5-dimensional metric ($A,B = 0,1,2,3,4$), y is the extra spatial coordinate, $\mathcal{R}$ is the 5-dimensional Ricci scalar, g is the trace of the 4-dimensional metric, R is the 4-dimensional Ricci scalar, and $\mathcal{L}_{\rm SM}$ is the Lagrangian of the fields in the Standard Model. The first term in Eq. (2) is the bulk 5-dimensional Einstein action, and the second term is the 4-dimensional Einstein action localized on the brane at $y = 0$. The induced metric on the brane is given by $g_{\mu\nu}(x) = g_{\mu\nu}^{(5)}(x, y = 0)$. The quantity $r_{\rm c}$ is the new parameter of the theory, and is the crossover scale which sets the scale for the transition from 4-dimensional to 5-dimensional gravity. For the maximally symmetric Friedmann-Robertson-Walker ansatz

$$ds_5^2 = f(y,H)ds_4^2 - dy^2, \qquad (3)$$

where ds_4^2 is the 4-dimensional maximally symmetric metric, and H is the 4-dimensional Hubble parameter, one gets a modified Friedmann equation on the brane of the form

$$H^2 \pm \frac{H}{r_{\rm c}} = \frac{8\pi G\rho_{\rm m}}{3}, \qquad (4)$$

where $\rho_{\rm m}$ is the matter density on the brane. This is often called the Friedmann equation of DGP gravity (Dvali, Gabadadze & Porrati 2000).

Inspired by the above example, one can also consider a generalized Friedmann equation

$$f(H) = H_0^2\Omega_{\rm m}(1+z)^3, \qquad (5)$$

where instead of modifying the matter content we consider modifications of gravity by having an arbitrary function f. Now assume that there is a critical scale, H_c, at which modifications start to have an effect. Such a scale will be close to the present Hubble parameter. At early times, when $H \gg H_c$, we know e.g. from nucleosynthesis constraints that $f(H) \approx H^2$. In general we can then expand f in terms of H_c/H:

$$H^2\sum_n c_n\left(\frac{H_c}{H}\right)^n = H_0^2\Omega_{\rm m}(1+z)^3. \qquad (6)$$

As long as $H \gg H_c$, evolution is standard and therefore $c_0 = 1$ and terms with $n < 0$ must vanish. Hence

$$H^2\left[1 + \sum_{n=1} c_n\left(\frac{H_c}{H}\right)^n\right] = H_0^2\Omega_{\rm m}(1+z)^3. \qquad (7)$$

Non-standard effects start to have an effect at late times i.e. when $H \sim H_c$. Expanding the sum gives

$$H^2\left[1 + c_1\frac{H_c}{H} + c_2\left(\frac{H_c}{H}\right)^2 + ...\right] = H_0^2\Omega_{\rm m}(1+z)^3, \qquad (8)$$

from which we see that the cosmological constant is the second order correction to the Friedmann equation. The first order correction corresponds to the DGP model.

Generally, the nth order correction for a flat universe is

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\rm m}(1+z)^3 + (1-\Omega_{\rm m})\left(\frac{H_0}{H}\right)^{n-2}, \qquad (9)$$

which is of the same form as (1) with $\alpha = 2 - n$.

In general, one can consider constraints on the different coefficients c_n with $n = 1, 2, ...$, much like is done in parametrizing dark energy (Alam et al. 2003). In this paper, we will for simplicity only consider a single term that is assumed to be the leading correction. The power of the correction is allowed to be arbitrary i.e. we do not restrict it to discrete values. This approach gives information on what the leading order term is, and gives an idea how well one constrain the different terms in the expansion with current and future data.

3 CONSTRAINTS FROM SUPERNOVAE TYPE IA

The first test any model attempting to explain the accelerated universe must pass is, of course, the SNIa data. We will in the following use the sample of 194 SNIa presented in Barris et al. (2004). The parameters we fit to the data are $\Omega_{\rm m}$ and α. We consider values $0 < \Omega_{\rm m} < 1$ and $-30 < \alpha < 2$. The upper limit on α comes from the limits on the amount of energy density present at the epoch of BBN (Dvali & Turner 2003). The Hubble parameter h is also involved in the fits, but is of little interest here, and we marginalize over it. The fit to the supernova data involves the luminosity distance $d_{\rm L} = c(1+z)\int_0^z dz/H(z)$, and we obtain $H(z)$ for given $\Omega_{\rm m}$ and α by solving Eq. (1) with a Newton-Raphson algorithm. The minimum χ^2 for the model was 195.7 for 191 degrees of freedom, with the best-fitting parameters $\Omega_{\rm m} = 0.56$, $\alpha = -14.8$. The two-parameter confidence contours for $\Omega_{\rm m}$ and α are shown in Fig. 1. It is clear that α is very weakly constrained by the supernova data, and can seemingly become arbitrarily large and negative. We next turn to the CMB data to see if they can provide firmer constraints.

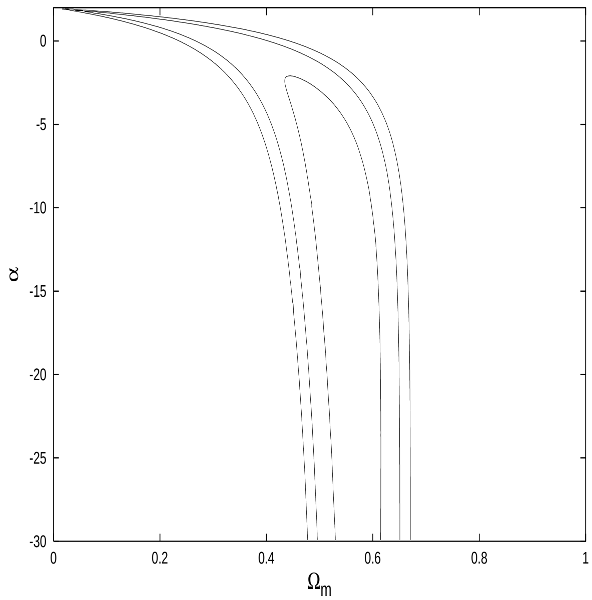

Figure 1. 68, 95 and 99% confidence contours for $\Omega_{\rm m}$ and α, resulting from fitting the collection of supernova data and marginalizing over h.

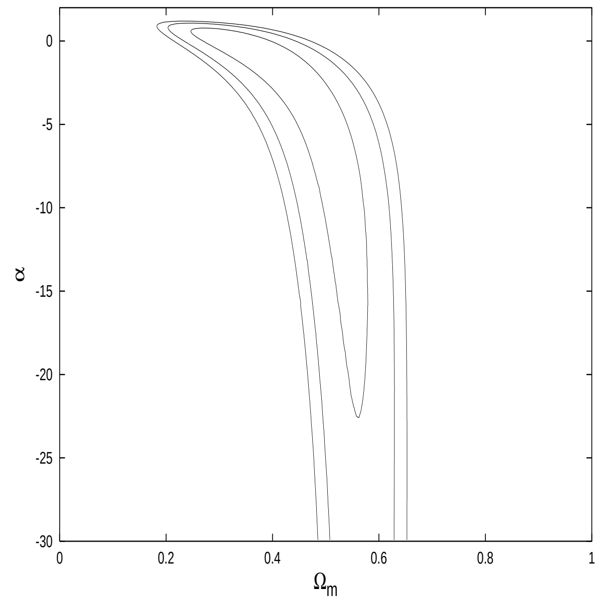

Figure 2. 68, 95 and 99% confidence contours for $\Omega_{\rm m}$ and α, resulting from fitting the collection of supernova data, and adding the constraint from the CMB shift parameter.

4 FITS TO CMB DATA

Before embarking on a full fit to the CMB data, we first consider the much simpler approach of just fitting the so-called CMB shift parameter

$$\mathcal{R} = \sqrt{\Omega_{\rm m}} H_0 r(z_{\rm dec}), \tag{10}$$

where $r(z) = \int_0^z dz/(H(z)/H_0)$ is the comoving distance in a flat universe, and $z_{\rm dec}$ is the redshift at decoupling. The shift parameter describes the shift in the CMB angular power spectrum when the cosmological parameters are varied (Bond, Efstathiou & Tegmark 1997; Melchiorri et al. 2002; Ödman et al. 2003). From WMAP, $z_{\rm dec} = 1088^{+1}_{-2}$, and $\mathcal{R}_{\rm obs} = 1.716 \pm 0.062$ (Spergel et al. 2003). Adding this constraint to the supernova fit is now straightforward (Wang & Mukherjee 2004). For each model, we compute $\mathcal{R}$ from Eq. (10) and add $\chi^2_{\mathcal{R}} = (\mathcal{R} - \mathcal{R}_{\rm obs})^2/\sigma^2_{\mathcal{R}}$, where $\sigma_{\mathcal{R}} = 0.062$ to the χ^2 for the supernova data. The resulting confidence contours in the $\Omega_{\rm m}$–α plane are shown in fig. 2. From the figure it is evident that adding the constraint coming from the shift parameter leads to a somewhat less degenerate range of α, especially when considering the 1σ contour. Still, even within the 1σ limits, α can be as small as -20. Such a small value corresponds to a universe that becomes dominated by the non-standard terms in the Friedmann equation very quickly after $H < H_0$.

The shift parameter is a useful tool for constraining cosmological models quickly and very easily using information from the CMB. However, it does not constrain cosmological parameters too well. For example, in the model considered in this paper, considering only the shift parameter gives a large degeneracy along the α axis, much like in the SNIa fit. Hence, in order to further constrain the parameter plane, one needs to consider the full shape of the CMB power spectrum.

When fitting the CMB data with a model based on an extra dimensional model, one should in principle start from the full extra dimensional theory. This can be problematic due to the bulk-brane interactions and hence we will in the following use a simplified approach where we solve the standard 4-d perturbed Boltzmann and fluid equations, but with the background evolution given by Eq. (1). For convenience, we will parametrize the effect of the extra term in the Friedmann equation by a dark energy component with an effective equation of state $w(a)$. As long as the extra component does not fluctuate, i.e. it only has an effect on the background evolution, such a parametrization is equivalent to modifying the Friedmann equation.

The effective equation of state is derived as follows. A dark energy component with equation of state $w(a)$ has a density which varies with the scale factor a according to

$$\rho_{\rm D}(a) = \rho_{0\rm D} \exp\left\{-3\int_1^a \frac{da'}{a'}[1+w(a')]\right\}, \tag{11}$$

so for a flat universe, $\Omega_{\rm D} = 1 - \Omega_{\rm m}$, the standard Friedmann equation is

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\rm m}}{a^3} + (1-\Omega_{\rm m}) \exp\left\{-3\int_1^a \frac{da'}{a'}[1+w(a')]\right\}. \tag{12}$$

Comparing this to eq. (1), we see that we must have

$$\exp\left\{-3\int_1^a \frac{da'}{a'}[1+w(a')]\right\} = \left(\frac{H}{H_0}\right)^\alpha, \tag{13}$$

for the two expressions to match. By taking the natural logarithm and differentiating with respect to a on both sides, we get

$$-3\frac{1+w(a)}{a} = \alpha\frac{d\ln(H/H_0)}{da}, \tag{14}$$

and since $da = a\,d\ln a$, we can write this as

$$w(a) = -1 - \frac{\alpha}{3}\frac{d\ln(H/H_0)}{d\ln a}. \tag{15}$$

Since $a = (1+z)^{-1}$, we can also write

$$\frac{d}{d\ln a} = -(1+z)\frac{d}{dz}, \tag{16}$$

giving

$$w(z) = -1 + \frac{\alpha}{3}\frac{(1+z)}{H/H_0}\frac{d(H/H_0)}{dz}. \tag{17}$$

Going back to eq. (1), we can differentiate with respect to z and get

$$\begin{aligned} 2\left(\frac{H}{H_0}\right)\frac{d(H/H_0)}{dz} &= \alpha(1-\Omega_{\rm m})\left(\frac{H}{H_0}\right)^{\alpha-1}\frac{d(H/H_0)}{dz} \\ &+ 3\Omega_{\rm m}(1+z)^2, \end{aligned} \tag{18}$$

so that

$$\frac{d}{dz}\left(\frac{H}{H_0}\right) = \frac{3\Omega_{\rm m}(1+z)^2}{2\left(\frac{H}{H_0}\right) - \alpha(1-\Omega_{\rm m})\left(\frac{H}{H_0}\right)^{\alpha-1}}. \tag{19}$$

This saves us the trouble of taking numerical derivatives in practical work, since $w(z)$ now can by expressed in terms of $H(z)/H_0$ as

$$w(z) = -1 + \frac{\alpha\Omega_{\rm m}(1+z)^3}{2\left(\frac{H}{H_0}\right)^2 - \alpha(1-\Omega_{\rm m})\left(\frac{H}{H_0}\right)^{\alpha}}. \tag{20}$$

So, instead of modifying the Friedmann equation directly, we can consider a standard Friedmann equation, Eq. (12), with a fluid whose equation of state is given by (20). Note that this is an exact reformulation of the background evolution. We have checked that fitting the supernova data with this reformulation results in confidence contours in agreement with those in Fig. 1.

For fitting the CMB TT power spectrum (Hinshaw et al. 2003; Kogut et al. 2003) we use the likelihood code provided by the WMAP team[1] (Verde et al. 2003). The CMB power spectra is computed by using CMBFAST code (Seljak & Zaldarriaga 1996), version 4.5.1[2]. For each point in the parameter space $(\Omega_{\rm m}, \alpha, h)$, we calculate the CMB TT power spectrum keeping the amplitude of the fluctuations a free parameter by finding the best-fitting amplitude for each set of parameters. In other words, we only fit the shape and not the amplitude of the power spectrum. In calculating the CMB power spectrum we use $\Omega_{\rm b} = 0.044$ (so that we vary the density of cold dark matter, $\Omega_{\rm c}$) and ignore reionization effects. Parameters $\Omega_{\rm m} \in [0, 1]$ and $\alpha \in [1.5, -10]$ have uniform priors and are chosen to cover the most interesting range of parameters. For the Hubble parameter h, we have explored different priors: a uniform prior $h \in [0.5, 1.0]$ and a Gaussian prior based on the HST Key Project value $h = 0.72 \pm 0.08$ (Freedman et al. 2001). We marginalize over h in making all the plots. The choice of prior makes little difference after marginalization since most of the weight comes from $h \sim 0.72$. Here we show results with a Gaussian prior but confidence contours with a uniform prior are essentially identical.

The WMAP TT power spectrum constraints are shown in fig. 3. From the figure we see that, as expected, having more information from the power spectrum than just the shift parameter helps tighten the constraints significantly.

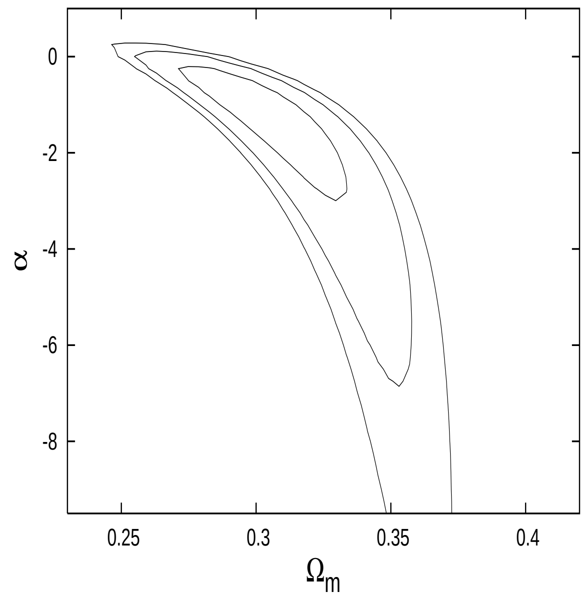

Figure 3. CMB constraints (68, 95, and 99 %) from the WMAP TT power spectrum

The minimum value of α within the 1σ contour is now only about -3. Comparing to fig. 1, where only SNIa are used, it is obvious that CMB provides a much tighter constraint to the model in question than just the SNIa results. This demonstrates how if one restricts oneself to a single cosmological probe in studying models with non-standard background evolution, the CMB can be a good first choice instead of supernovae. If the non-standard model involves new cosmological fluctuating fluids, then a simple check is provided by considering LSS observations which can be effective in constraining fluids with a non-zero sound speed.

The combined fit CMB+SNIa is shown in fig. 4. Adding the SNIa data further relieves the degeneracy along the α axis due to the fact even though both fig. 1 and 3 are both somewhat degenerate along α, the region of degeneracy correspond to different values of $\Omega_{\rm m}$.

5 FUTURE SUPERNOVA DATA

We have found that the present supernova data cannot put significant constraint on the parameter α, and that at the present time the most stringent constraints comes from the combination of CMB data with supernovae. An interesting question is how well one can do with future SNIa surveys. The planned Dark Energy Probe [3]/ Supernova Acceleration Probe[4] is expected to observe about 2000 supernovae type Ia per year out to a redshift of $z \sim 1.7$ (Aldering et al. 2002), and this should improve the power of this probe to constrain dark energy models considerably. We will in the following simulate data sets of this type, following the approach of Saini, Weller & Bridle (2004), and consider how

[1] http:// lambda.gsfc.nasa.gov/
[2] http://www.cmbfast.org/
[3] http://universe.gsfc.nasa.gov/program/darkenergy.html
[4] http://snap.lbl.gov/

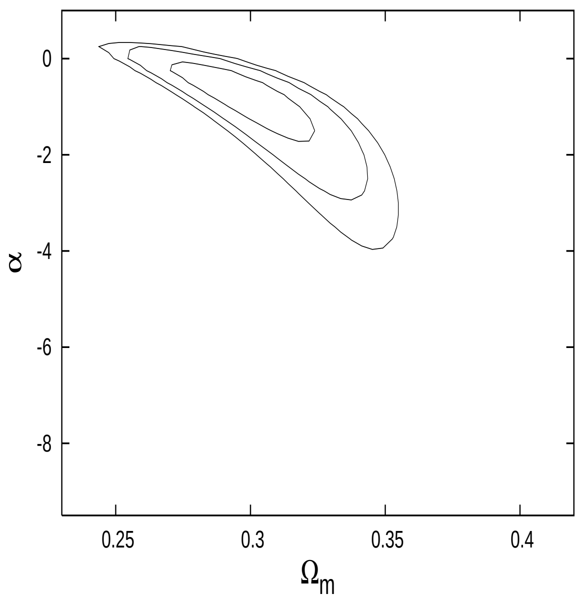

Figure 4. Constraints (68, 95, and 99 %) from combining the WMAP TT power spectrum with supernova Type Ia data.

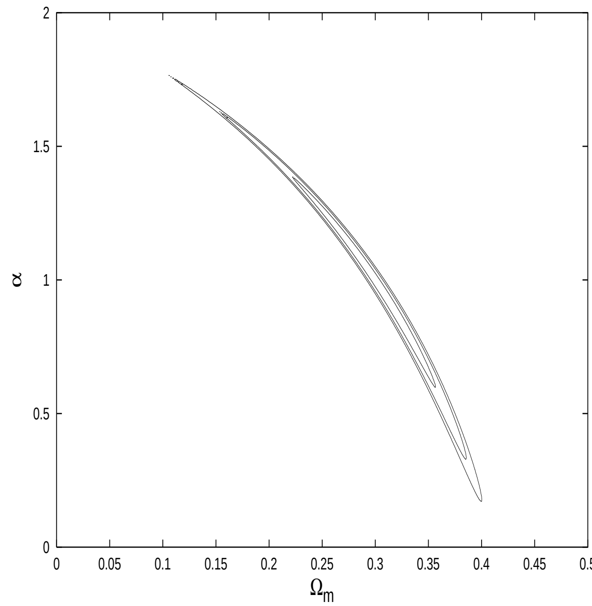

Figure 5. Constraints on Ω_m and α from simulated data based on $\Omega_\mathrm{m} = 0.3$, $\alpha = 1$.

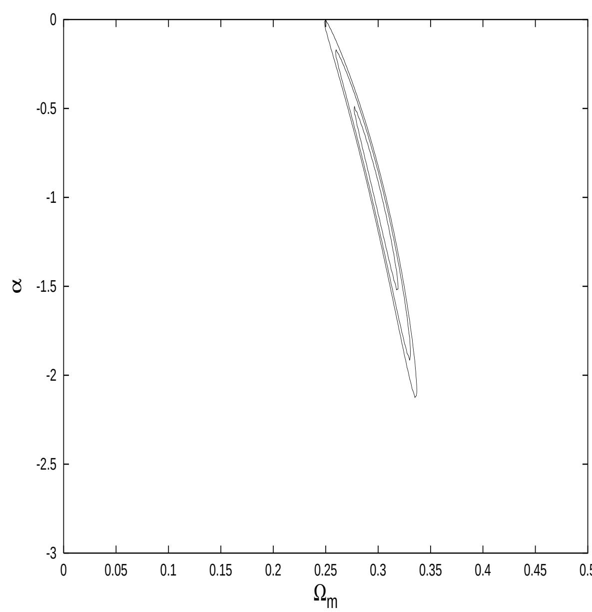

Figure 6. Constraints on Ω_m and α from simulated data based on $\Omega_\mathrm{m} = 0.3$, $\alpha = -1$.

well they can constrain the the model under investigation in this paper.

Empirically, SNIa are very good standard candles with a small dispersion in apparent magnitude $\sigma_\mathrm{mag} = 0.15$, and there is no indication of redshift evolution. The apparent magnitude is related to the luminosity distance through

$$m(z) = \mathcal{M} + 5 \log D_\mathrm{L}(z), \tag{21}$$

where $\mathcal{M} = M_0 + 5 \log[(c/H_0)\,\mathrm{Mpc}^{-1}] + 25$. The quantity M_0 is the absolute magnitude of type Ia supernovae, and $D_\mathrm{L}(z) = H_0 d_\mathrm{L}(c)/c$ is the Hubble constant free luminosity distance. The combination of the absolute magnitude and the Hubble constant, $\mathcal{M}$, can be calibrated by low redshift supernovae (Hamuy et al. 1993; Perlmutter et al. 1999). The dispersion in the magnitude, σ_mag, is related to the deviation in the distance, σ, by

$$\frac{\sigma}{d_\mathrm{L}(z)} = \frac{\ln 10}{5} \sigma_\mathrm{mag}. \tag{22}$$

In our simulated data sets, we assume that the errors in the luminosity distance are Gaussian and given by Eq. (22). We neglect systematic errors. Furthermore, we assume that the supernovae type Ia are uniformly distributed and bin them in 50 redshift bins, giving a relative error in the luminosity distance in each bin of $\sim 1\%$. We do not add noise to the simulated d_L, and hence our results give the ensemble average of the parameters we fit to the simulated data sets.

First, we simulate a data set based on a model with $\Omega_\mathrm{m} = 0.3$, $\alpha = 1$. In Fig. 5 we show the confidence contours resulting from fitting Ω_m and α to this simulated data set. The constraints which can be derived from a data set of this quality are seen to be considerably tighter than those derived from the presently available data. However, there is still a notable degeneracy between Ω_m and α, so that without a tight prior on Ω_m, one cannot distinguish between, e.g., $\alpha = 1$ and $\alpha = 0.5$. A similar situation occurs for simulated data based on $\Omega_\mathrm{m} = 0.3$, $\alpha = -1$, shown in Fig. 6. For this case, we also show in Fig. 7 the marginalized, normalized probability distributions for α for three different priors on Ω_m: a uniform prior $0 < \Omega_\mathrm{m} < 0.5$, a Gaussian prior $\Omega_\mathrm{m} = 0.30 \pm 0.02$, and a Gaussian prior $\Omega_\mathrm{m} = 0.300 \pm 0.005$. Note that only with the last choice of prior, where Ω_m is assumed to be known to within 1.7 %, does one get a really tight constraint on α, but that even in the case of a uniform prior one can exclude a cosmological constant (which corresponds to $\alpha = 0$) at 99.9 % confidence.

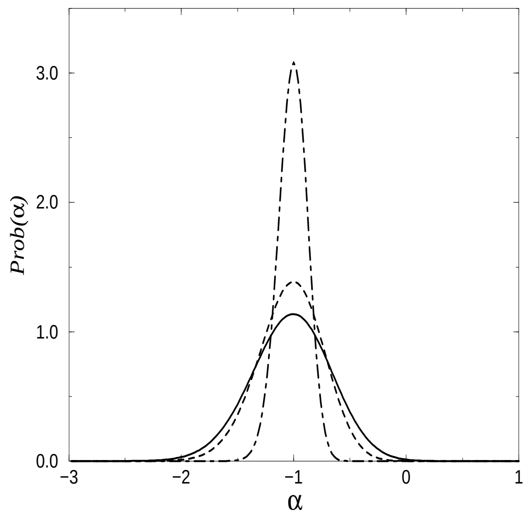

Figure 7. Marginalized probability distribution for α with three different choices of priors on $\Omega_{\rm m}$: uniform $0 < \Omega_{\rm m} < 0.5$ (solid line), Gaussian $\Omega_{\rm m} = 0.30 \pm 0.02$ (dashed line), and Gaussian $\Omega_{\rm m} = 0.300 \pm 0.005$ (dot-dashed line).

6 CONCLUSIONS

In this work we have studied observational constraints on a modified Friedmann equation that mimics dark energy in the universe. Modifications of the type we have considered here may occur naturally in models with large extra dimensions, and provide an attractive alternative to introducing an unknown component with negative pressure. We have found that the combination of the magnitude-redshift relationship derived from supernovae type Ia and the WMAP TT power spectrum constrain the exponent of the extra term in the modified Friedmann equation, $(H/H_0)^\alpha$, to be around -1. From the point of the expanded Friedmann equation, this corresponds to $n = 3$, which is the next term after the cosmological constant in the expansion (7). In particular, a cosmological constant, corresponding to $\alpha = 0$ or $n = 2$ seems to be disfavoured for the studied parameter space. Furthermore, the first order correction $n = 1$ corresponding to the DGP model appears to be strongly disfavored over the $n = 3$ term.

We have also used simulated data sets of the type one can expect from future satellite-based supernova surveys to estimate the accuracy with which one can hope to constrain the corrections to the Friedmann equation. Tight constraints on α can be expected if the matter density parameter $\Omega_{\rm m}$ is known accurately from other observations. But even without any priors on $\Omega_{\rm m}$, one can rule out a cosmological constant at high significance if the true universe is described by $\alpha = -1$.

An obvious omission in this work is that we have not considered constraints coming from LSS. Since there is no extra negative pressure fluid in the model, one does not expect large deviations in the matter power spectrum. Furthermore, the background evolution follows the standard behaviour until very recently, which suggests that linear growth will be standard for the most of the expansion history (Lue, Scoccimarro & Starkman 2004; Multamäki, Gaztañaga & Manera 2003). A more detailed analysis of linear and non-linear growth is left for future work.

Finally, we note that the term we have considered is only the leading order correction to the standard Friedmann equation. The present data indicate that the first order correction is (H_0/H) but other terms can also play a role. It is an interesting question whether a combination of data sets of the quality we can expect in the future can constrain the number of correction terms and their form.

ACKNOWLEDGMENTS

ØE gratefully acknowledges the hospitality of NORDITA, where parts of this work were carried out. TM is grateful to the Institute of theoretical astrophysics, University of Oslo, for their hospitality during the first stages of this work.

REFERENCES

Alam U., Sahni V., Saini T. D., Starobinsky A. A., 2003, MNRAS, 344, 1057
Aldering G., the SNAP collaboration, 2002, in Dressler A. M., ed., Proc. SPIE 4835, Future Research Direction and Visions for Astronomy. Int. Soc. Opt. Eng., Bellingham, WA, p. 146
Barris B. J. et al., 2004, ApJ, in press, preprint (astro-ph/0310843)
Bean R., Dore O., 2003, Phys. Rev. D, 68, 023515
Bilic N., Tupper G. B., Viollier R., 2001, Phys. Lett. B, 535, 17
Bond J. R., Efstathiou G., Tegmark M., 1997, MNRAS, 291, L33
Deffayet C., 2001, Phys.Lett B, 502, 199
Deffayet C., Dvali G., Gabadadze G., 2002, Phys. Rev. D, 65, 044023.
Dvali G., Gabadadze G., Porrati M., 2000, Phys.Lett. B, 485, 208
Dvali G., Turner M. S., 2003, preprint (astro-ph/0301510)
Efstathiou G. et al., 2002, MNRAS, 330, L29
Freedman W.L. et al., 2001, ApJ, 553, 47
Freese K., Lewis M., 2002, Phys. Lett. B, 540, 1
Hamuy M. et al., 1993, ApJ, 106, 2392.
Hinshaw G. et al., 2003, ApJS, 148, 135
Kamenshchik A., Moschella U., Pasquier V., 2001, Phys. Lett. B, 511, 265
Kogut A. et.al., 2003, ApJS, 148, 161
Lue A., Scoccimarro, R., Starkman G.D., 2004, preprint (astro-ph/0401515)
Multamäki T., Gaztañaga E., Manera M., 2003, MNRAS, 344, 761
Melchiorri A., Mersini L., Ödman C. J., Trodden M., 2002, preprint (astro-ph/0211522)
Peebles P. J. E., Ratra, B., 1988, ApJ, 325, L17
Perlmutter S. et al., 1999, ApJ, 517, 565
Ratra B., Peebles P. J. E., 1988, Phys. Rev. D, 37, 3406
Riess A. G. et al., 1998, AJ, 116, 1009
Saini T. D., Weller J., Bridle, S. L., 2004, MNRAS, 348, 603

Sandvik H., Tegmark M., Zaldarriaga M., Waga I., 2002, preprint (astro-ph/0212114)
Seljak U., Zaldarriaga M., 1996, ApJ, 469, 437
Solheim J.-E., 1966, MNRAS, 133, 321
Spergel D. N. et al., 2003, ApJS, 148, 175
Tegmark M. et al., 2003, preprint (astro-ph/0310723)
Verde L. et al., 2003, ApJS, 148, 195
Wang Y., Mukherjee P., 2004, ApJ, in press, preprint (astro-ph/0312192)
Wetterich C., 1988, Nucl. Phys. B, 302, 668
Ödman C. J., Melchiorri A., Hobson M. P., Lasenby A. N., 2003, Phys. Rev. D, 67, 083511